

# **Quantum Non-Markovianity**

**2022**

**Talk**

**Abstracts**

## Characterizing Non-Markovian Quantum Computers

Robin Blume-Kohout

“Markovian”, and its evil cousin “non-Markovian”, are surprisingly subjective. First, the operational and conceptual meaning of the adjective “Markovian” varies depending on exactly what kind of thing it’s being applied to. A Markovian stochastic process and a Markovian quantum computer satisfy distinct conditions. Second, whether a system’s behavior is Markovian depends not just on the nature of the system, but on the state space used to model it. Although these points feel kind of pedantic, they turn out to have big (and confusing) impacts on how “non-Markovian” is used to describe quantum computing hardware! In this talk, I’ll explain exactly what “Markovian” usually means in QCVV (quantum characterization, verification and validation) of real-world quantum computers, and then I’ll explore some of the more subtle complexities raised by those two subjectivities. I’ll speculate about the practical consequences of non-Markovianity in quantum computers, and present some of the tools we use to detect and quantify it. I’ll conclude by sketching out how we hope to characterize and (efficiently) model common non-Markovian errors in next-generation quantum processors.

SNL is managed and operated by NTESS under DOE NNSA contract DE-NA0003525.

# On the relation between time local and non-local master equations

Nina Megier

Any realistic physical system is unavoidably coupled to some external degrees of freedom and should then be treated as an open system. In the study of the dynamics of an open quantum system it is well known that alternative approximated descriptions are often useful. Here we show that also equivalent exact evolution equations can be introduced, investigating a newly introduced link between time local and time non-local master equations [1, 2]. Though these equations are in principle equivalent as their solutions are the same reduced density operator, the knowledge of both can be beneficial. Our approach, based on the damping basis representation, give some insights into the occurrence of different dephasing channels in time local and time non-local master equations and the non-Markovianity property of the exact and the approximated reduced dynamics.

[1] N. Megier, A. Smirne, B. Vacchini, *New J. Phys.* 22: 083011, 2020

[2] N. Megier, A. Smirne, B. Vacchini, *Entropy* 22(7): 796, 2020

# Tackling the challenge of simulating non-Markovian open quantum systems.

Nicola Lorenzoni

Outside the range of validity of the Born, Markov and secular approximations, simulating the dynamics of an open quantum system results in a challenge that often becomes practically intractable.

In the present talk, we consider model systems coupled non-perturbatively to highly structured environments. Due to the complexity of the system-environment coupling, approximated methods, as involving a markovian treatment, have been widely used.

In here, we inquire the capability of approximated methods to describe the spectral response of these systems, and finally we propose and discuss some of the state of the art methodologies capable of exactly simulating the dynamics of an open quantum systems.

## What is Quantum (non)Markovianity?

Howard Wiseman

co-authors: Li (Kenny) Li, and Michael Hall [1]

Markovianity versus non-Markovianity is a well-established distinction for classical systems. The same cannot be said for quantum systems. Different communities and individuals use “quantum Markovianity” to mean very different things. We argue that, to avoid confusion, it may be best to avoid attributing that term any definite meaning. However, that does not mean that there is nothing to say about Markovianity for open quantum systems. We discuss a large number of concepts that have been, or could logically be, used to distinguish quantum Markovianity, and prove hierarchical relations between them. Some are existing concepts, including “factorisation”, “quantum regression formula”, “divisibility”, and “Lindblad”. Others we introduce, including “past-future independence”, and “composability”. We also prove relations between these and other properties of interest for open quantum systems, such as the applicability of dynamical decoupling to preserve quantum information, the existence of (quantum) information backflow from the environment, and the physical reality of stochastic pure-state trajectories. Finally, we discuss in which concept the closest analogue of classical Markovianity lies.

[1] Li Li, Michael J.W. Hall, and Howard M. Wiseman, “Concepts of quantum non-Markovianity: A hierarchy” [Physics Reports](#) **759**, 1-51 (2018). DOI: [10.1016/j.physrep.2018.07.001](https://doi.org/10.1016/j.physrep.2018.07.001)

## Non-Markovian noise in donor spin qubit devices in silicon.

H.G. Stemp<sup>a</sup>, S. Asaad<sup>a</sup>, R. Savytsky<sup>a</sup>, T. Botzem<sup>a</sup>, S.Freer<sup>a</sup>, M.A.I. Johnson<sup>a</sup>, A.Laucht<sup>a</sup>  
and A. Morello<sup>a</sup>.

<sup>a</sup> *School of Electrical Engineering and Telecommunications, UNSW Sydney, Sydney, NSW  
2052, Australia*

The spins of ion-implanted donor atoms in silicon have shown excellent promise as a platform to host a quantum processor, due to their record coherence times and high fidelity 1 and 2-qubit operations [1][2][3]. The long-term viability of this platform will benefit from having a microscopic understanding of the noise sources that influence the operation fidelity of the qubits. One prevalent noise source observed in these devices is an effect known as ‘pulse-induced frequency shift’. This is a non-Markovian effect, whereby the electron spin resonance frequency shifts, over some finite timescale, as a result of the application of the driving fields required to control the qubits. Although this pulse-induced frequency shift represents a significant source of error when operating these devices, the microscopic origins of this effect are yet to be understood. Here we show experimental data we acquired to characterise and understand this non-Markovian noise source, as well as the present and future techniques we are implementing to mitigate this effect.

- [1] Muhonen, J. T. et al. *Storing quantum information for 30 seconds in a nanoelectronic device*. Nature nanotechnology 9, 986 (2014).
- [2] Pla, J.J. et al. *High fidelity readout and control of a nuclear spin qubit in silicon*. Nature 496, 334-338 (2013).
- [3] Mądzik, M. T. et al. *Conditional quantum operation of two exchange-coupled single-donor spin qubits in a MOS-compatible silicon device*. Nature Communications 12, 181 (2021).

# Role of quantum divergences in non-Markovian dynamics

Bassano Vacchini

An interesting approach for the description of memory effects in the reduced quantum dynamics of an open system is based on the notion of information exchange between the open system and its environment. This exchange has typically been quantified studying the variation in time of the trace distance between distinct initial system states. We point to the fact that such an information exchange can actually be described by a large class of quantum divergences, including not only distances, but also entropic quantifiers. We consider in particular regularized versions of quantum relative entropy. We derive general upper bounds on the revivals of quantum divergences conditioned and determined by the formation of correlations and changes in the environment. We further consider the role of these developments for the description of memory properties in terms of divisibility properties.

# Stochastic unravellings of master equations: Designing quantum-jump realizations

Jyrki Piilo

Stochastic methods are commonly used to unravel master equations and to solve open quantum systems dynamics both in Markovian and non-Markovian regime. For non-Markovian dynamics with memory effects, one can use, e.g., non-Markovian quantum jumps [1] while recently introduced rate operator quantum jump method allows to treat any dynamical regime [2]. Moreover, a connection between jump and diffusion descriptions in non-Markovian regime was also found recently [3]. In terms of the rate operator approach [2], we show that, as a matter of fact, it opens the path to a wide range of different unravelings which in turn leads to a large freedom in designing the quantum-jump realizations [4]. These results bear significant importance also for the measurement scheme interpretation of open system dynamics and allow, e.g., to shift the unitary drive of open system to its quantum jump part within the unravelling.

On another theme and time allowing, we discuss the concept of reverse decoherence, hidden nonlocality, and efficient teleportation under noise.

[1] J. Piilo, S. Maniscalco, K. Harkonen, and K.-A. Suominen, *Phys. Rev. Lett.* 100, 180402 (2008).

[2] A. Smirne, M. Caiaffa, and J. Piilo, *Phys. Rev. Lett.* 124, 190402 (2020).

[3] K. Luoma, W. T. Strunz, and J. Piilo, *Phys. Rev. Lett.* 125, 150403 (2020).

[4] D. Chruscinski, K. Luoma, J. Piilo, and A. Smirne, *Quantum* 6, 835 (2022).

[5] Z.-D. Liu, O. Siltanen, T. Kuusela, R.-H. Miao, C.-X. Ning, C.-F. Li, G.-C. Guo, and J. Piilo, *arXiv:2210.14935* (2022).



# Predicting and controlling non-Markovian quantum dynamics

Gerardo Paz Silva

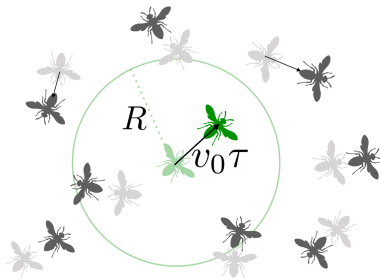
One of the key tasks in the development of quantum technologies is predicting and eventually controlling the behaviour of a general open quantum system for long times. Often, if not always, predicting the behaviours of system and bath is impossible, and so one is restricted to studying the reduced dynamics of the system. This represents a loss of information and is the main difficulty in long time analysis. To mitigate this problem, we introduce a technique which keeps tracks not only of the evolution of the system but also of (measurable) bath-related quantities which influence the dynamics of the quantum system of interest. This allows us to predict the behaviour of the system for longer times, as compared to existing tools with the same seed information, e.g., with the same perturbative order. Finally, we show how our technique allows us to (in principle) exactly track the evolution of high order correlations of the dephasing spin boson model even when the state is non-Gaussian.

# The role of memory effects in classical nonequilibrium systems

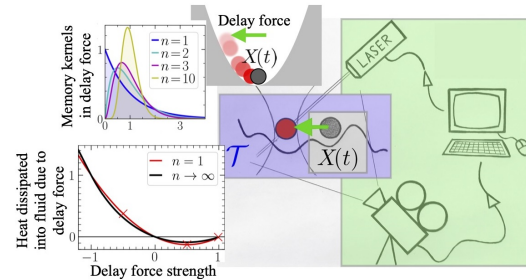
Sarah A.M. Loos

*Department of Applied Mathematics and Theoretical Physics  
University of Cambridge  
Cambridge, United Kingdom*

*sl2127@cam.ac.uk*



Finite detection and reaction times of agents give rise to time-delayed interactions [5].



Feedback control on mesoscale object gives rise to stochastic dynamics with time delay [2,3].

One of the great challenges of statistical physics is the extension towards systems that operate far from thermal equilibrium and therefore do not fall into the usual paradigm of the classical theories. For example, are there common physical laws that govern the dynamics of molecular machines, flocks of bird, or even computer systems? In recent years, the framework of stochastic thermodynamics has led to considerable advances in the understanding of universal mechanism that govern such nonequilibrium systems.

However, many of the approaches in statistical physics and stochastic thermodynamics are fundamentally based on the Markov assumption, while in reality such complex nonequilibrium systems typically exhibit non-negligible memory effects. Memory effects can emerge due to an incomplete time-scale separation between ‘system’ and ‘bath’ degrees of freedom, due to finite perception and reaction delays of biological systems, or as an implicit consequence of the presence of control loops.

In this talk, I will give an overview of theoretical challenges and different approaches to deal with the occurrence of non-Markovian dynamics in classical systems [1-5]. As particular examples, I will discuss the physical implications of time delay in feedback control on mesoscale objects [2,3] and in the collective motion of swarms of insects [5].

- [1] S.A.M. Loos, S.H.L. Klapp: Fokker–Planck Equations for Time-Delayed Systems via Markovian Embedding, *J. Stat. Phys.* **177**, 95–118 (2019).
- [2] S.A.M. Loos, S. Hermann, S.H.L. Klapp: Medium Entropy Reduction and Instability in Stochastic Systems with Distributed Delay, *Entropy* **23**, 696 (2021).
- [3] S.A.M. Loos, S.H.L. Klapp: Heat flow due to time-delayed feedback, *Sci. Rep.* **9**, 1-11 (2019).
- [4] T. Doerries, S.A.M. Loos, S.H.L. Klapp: Correlation functions of non-Markovian systems out of equilibrium: Analytical expressions beyond single-exponential memory, *J. Stat. Mech.* **3**, 033202 (2021).
- [5] V. Holubec, D. Geiss, S.A.M. Loos, K. Kroy, F. Cichos: Finite-size scaling at the edge of disorder in a time-delay Vicsek model, *Phys. Rev. Lett.* **127**, 258001 (2021).

# Resource-Efficient Non-Markovian Characterization & Control with Frames

Lorenza Viola

Accurate characterization and control of open quantum systems subject to realistic, non-Markovian noise are vital for exploiting the full potential of quantum technologies and eventually reaching quantum fault tolerance. I will describe how to leverage the mathematical notion of a “frame” to build a simplified, “model-reduced” representation of the noisy dynamics of interest, which is directly tied to the available, finite control resources. Such a frame-based, “control-adapted” formulation overcomes important limitations of existing approaches. In particular, I will outline how it allows to implement hardware-efficient digital quantum noise spectroscopy directly in the time domain, and report progress toward achieving optimal quantum gate design under non-Gaussian noise.

# Noncommuting charges and their effect on entanglement

Shayan Majidy, Nicole Yunger Halpern

In conventional thermodynamics, a system of interest and an environment exchange quantities—energy, particles, electric charge, etc.—that are globally conserved and are represented by Hermitian operators. These operators were implicitly assumed to commute with each other, until a few years ago. Freeing the operators to fail to commute has enabled many theoretical discoveries. For example, noncommuting charges were shown to reduce entropy-production rates and may enhance finite-size deviations from eigenstate thermalization. This talk overviews noncommuting thermodynamic charges and then highlights a recent technical result—noncommuting charges can increase entanglement.

# Tensor network based machine learning of non-Markovian quantum processes

Dario Poletti

Tensor networks have been used to study many-body quantum systems for about two decades. Recently it has been shown that they can also be used as a linear network for machine learning tasks. We show how to learn structures of generic, non-Markovian, quantum stochastic processes using a tensor network based machine learning algorithm. We do this by representing the process as a matrix product operator (MPO) and train it with a database of local input states at different times and the corresponding time-nonlocal output state. In particular, we analyze a qubit coupled to an environment and predict output state of the system at different time, as well as reconstruct the full system process. We show how the bond dimension of the MPO, a measure of non-Markovianity, depends on the properties of the system, of the environment and of their interaction. Hence, this study opens the way to a possible experimental investigation into the process tensor and its properties.

Journal ref: Phys. Rev. A 102, 062414 (2020)

# Tensor Network Methods for Quantum Systems Coupled to Gaussian Environments

Aidan Strathearn

Developing scalable quantum technologies requires an intricate understanding of the effects of environmental noise on quantum systems. In the weak coupling or Markovian regimes of such open quantum systems, where the environment is approximated as being ‘memoryless’, Lindblad master equations are typically successful in describing the salient relaxation and decoherence processes. However, general open quantum systems can be strongly coupled to their environments, whose influence is generally non-locally correlated in time. Such non-Markovian systems can be difficult to describe analytically and numerical methods are often required to understand them.

In this talk I will describe a numerically exact and efficient methodology for modelling the class of non-Markovian quantum systems whose environment is Gaussian. Gaussian environments can be integrated over exactly, be they fermionic, bosonic, quantum or classical. The description of the reduced system is then given in terms of an influence functional which is non-local in time and which can be represented as a tensor network [1]. This tensor network is essentially the process tensor for the system [2], from which all observables of the system can then be found. The general methodology has found widespread application. For example, in studying: dissipative quantum phase transitions [3], optimal control of open systems [4], many-body non-Markovian dynamics [5, 6], and quantum thermodynamics [7]. In the talk I will discuss recent applications of the method to systems with non-additive environments [8], and in calculating mean-force Gibbs states [9].

## References

- [1] Aidan Strathearn. *Modelling non-Markovian quantum systems using tensor networks*. Springer International Publishing, 2020.
- [2] Mathias R. Jørgensen and Felix A. Pollock. Exploiting the causal tensor network structure of quantum processes to efficiently simulate non-markovian path integrals. *Phys. Rev. Lett.*, 123:240602, Dec 2019.
- [3] A. Strathearn, P. Kirton, D. Kilda, J. Keeling, and B. W. Lovett. Efficient non-Markovian quantum dynamics using time-evolving matrix product operators. *Nat. Commun.*, 9(1):3322, aug 2018.
- [4] Gerald E Fux, Eoin P Butler, Paul R Eastham, Brendon W Lovett, and Jonathan Keeling. Efficient exploration of Hamiltonian parameter space for optimal control of non-Markovian open quantum systems. *Phys. Rev. Lett.*, 126(20):200401, 2021.
- [5] Gerald E. Fux, Dainius Kilda, Brendon W. Lovett, and Jonathan Keeling. Thermalization of a spin chain strongly coupled to its environment, 2021.
- [6] Piper Fowler-Wright, Brendon W. Lovett, and Jonathan Keeling. Efficient many-body non-markovian dynamics of organic polaritons. *Phys. Rev. Lett.*, 129:173001, Oct 2022.
- [7] Maria Popovic, Mark T. Mitchison, Aidan Strathearn, Brendon W. Lovett, John Goold, and Paul R. Eastham. Quantum heat statistics with time-evolving matrix product operators. *PRX Quantum*, 2:020338, Jun 2021.
- [8] Dominic Gribben, Dominic M. Rouse, Jake Iles-Smith, Aidan Strathearn, Henry Maguire, Peter Kirton, Ahsan Nazir, Erik M. Gauger, and Brendon W. Lovett. Exact dynamics of nonadditive environments in non-markovian open quantum systems. *PRX Quantum*, 3:010321, Feb 2022.
- [9] Yiu-Fung Chiu, Aidan Strathearn, and Jonathan Keeling. Numerical evaluation and robustness of the quantum mean-force gibbs state. *Phys. Rev. A*, 106:012204, Jul 2022.

# Scalably and self-consistently learning the many-time physics of quantum stochastic processes

Gregory White

The demands of fault tolerance mean that a wide variety of simple and exotic noise types must be tamed for quantum devices to progress. Crucially, this means keeping up with complex correlated — or non-Markovian — effects, both with respect to the background process and to control operations. Recently, we have developed a generalised version of quantum process tomography to characterise arbitrary non-Markovian processes in practice. The resulting estimate contains all information about the multi-time correlations, and carries straightforward interpretations that can be brought across from many-body physics. But the technique is both expensive and relies on known control. To scale and generalise, we develop different methods to efficiently reconstruct both temporal and spatiotemporal tensor network multi-time process representations. These are not only efficient, but expressive enough to incorporate fully generic control errors. Our approach to characterise these components therefore employs no assumptions about prior calibrations, and can accommodate large numbers of time-steps and qubits from relatively few experiments. The result is a practical, scalable, and self-consistent procedure capable of describing arbitrary open dynamics and experimental controls. We bolster these claims both numerically and experimentally, demonstrating how to access and learn the many-time phenomena exhibited by non-Markovian open quantum systems. The framework is not only useful for diagnostic purposes, but applicable at all levels of the quantum control stack: in error suppression, error mitigation, and error correction.

# Tensor networks for quantum non-Markovian collisional dynamics

Sergey Filippov

Quantum collision models are receiving increasing attention as they describe many nontrivial phenomena in the dynamics of open quantum systems [1,2]. In a general scenario of both fundamental and practical interest, a quantum system repeatedly interacts with individual particles or modes, forming a correlated and structured reservoir; however, classical and quantum environment correlations greatly complicate the calculation and interpretation of the system dynamics. We propose an exact solution to this problem based on the tensor network formalism [3]. We find a natural Markovian embedding for the system dynamics, where the role of an auxiliary system is played by virtual indices of the network. The constructed embedding is amenable to an analytical treatment for a number of timely problems such as the system interaction with two-photon wave packets, structured photonic states, and one-dimensional spin chains. We also derive a time-convolution master equation and relate its memory kernel with the environment correlation function, thus revealing a clear physical picture of memory effects in the dynamics. The results advance tensor-network methods in the fields of quantum optics and quantum transport. Higher-order stroboscopic limits for the collisional dynamics and a transition from non-Markovian to Markovian regime (even if the environment is correlated) is discussed too [4].

[1] F. Ciccarello, S. Lorenzo, V. Giovannetti, and G. M. Palma, Quantum collision models: Open system dynamics from repeated interactions, *Phys. Rep.* 954, 1 (2022).

[2] S. Campbell and B. Vacchini, Collision models in open system dynamics: A versatile tool for deeper insights? *Europhys. Lett.* 133, 60001 (2021).

[3] S. N. Filippov, I. A. Luchnikov. Collisional open quantum dynamics with a generally correlated environment: Exact solvability in tensor networks. *Phys. Rev. A* 105, 062410 (2022).

[4] S. N. Filippov. Multipartite correlations in quantum collision models. *Entropy* 24, 508 (2022).



# Witnessing quantum memory in non-Markovian processes

Christina Giarmatzi<sup>a,b</sup> and Fabio Costa<sup>b</sup>

<sup>a</sup>*Centre for Quantum Software and Information, University of Technology Sydney, NSW 2007, Australia.*

<sup>b</sup>*Centre for Engineered Quantum Systems, School of Mathematics and Physics, University of Queensland, QLD 4072 Australia.*

The study of open quantum systems is a vast field within quantum physics that concerns the interaction between some system and its environment. Its great importance lies on the fact that every experimental realisation of a quantum process faces the possibility of noise coming from the environment. Although in small quantum devices the noise is assumed to be uncorrelated (Markovian) this assumption fails as the size and complexity increases, and the various system-environment interactions become correlated (non-Markovian).

We present a method to detect quantum memory in a non-Markovian process. We call a process Markovian when the environment does not provide a memory that retains correlations across different system-environment interactions. We define two types of non-Markovian processes, depending on the required memory being classical or quantum. We formalise this distinction using the process matrix formalism [1], through which a process is represented as a multipartite state. Within this formalism, a test for entanglement in the state can be mapped to a test for quantum memory in the corresponding process. This allows us to apply separability criteria and entanglement witnesses to the detection of quantum memory.

We demonstrate the method in a simple model: system and environment are qubits jointly prepared in a maximally entangled state and later interact according to the Ising model, in between two measurement stations A and B for the system. A quantum memory witness corresponds to a set of operations at A and B. As separability criteria for the search of witnesses we use the positive partial transpose (PPT) applied on the state [2] and on symmetric extensions of the state [3]. To find a witness, we cast each criterion as a SemiDefinite Program that can be solved efficiently. This also allows us to restrict the search for witnesses, possibly tailored to experimental limitations. As with entanglement witnesses, our method of witnessing quantum memory provides a versatile experimental tool for open quantum systems.

[1] O. Oreshkov, F. Costa, and Č. Brukner, *Nat. Commun.* **3**, 1092 (2012).

[2] A. Peres, *Physical Review Letters* **77**, 1413 (1996).

[3] A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, *Physical Review A* **69**, 022308 (2004).

# Experimental multi-time process tomography on superconducting qubits

Tyler Jones

The maturation of quantum technologies in the past decade has been enabled, in no small part, by the ongoing advancement of noise characterisation techniques. For Markovian noise sources, quantum process tomography or QPT (a natural extension of quantum state tomography) is one such technique which has become ubiquitous in the experimental toolbox. However, techniques like QPT have no capacity to characterise sources of non-Markovian noise (e.g. memory effects, system drifts and crosstalk), which we know exist nontrivially in the vast majority of experimental hardware. In this talk, we discuss a methodology to perform complete multi-time quantum process tomography, which allows for characterisation of non-Markovian dynamics. We discuss the experimental procedure required to perform multi-time process tomography on a superconducting quantum device, and present results where we successfully detect quantum memory effects on an in-house superconducting device and a publicly available IBM processor.